Natural Computation and Self-Organization:
The Physics of Information Processing in Complex Systems
Winter 2008
Syllabus

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Time: TuTh 2:10 - 3:30 PM
Location: 158 Roessler Hall
WWW: `http://cse.ucdavis.edu/~chaos/courses/ncaso/`

Contents

First Lecture (8 January): Overview

Readings (available via course website):

- *Chaos*, JP Crutchfield, JD Farmer, NH Packard, RS Shaw, Scientific American **255** (1986) 46–57.
- *Odds*, Stanislaw Lem, New Yorker **54** (1978) 38–54.

Topics:

1. Introduction and motivations
2. Four parts: Self-Organization, Measurement Theory, Information Processing, Natural Computation
3. Survey interests, background, and abilities
4. Course logistics
5. Exams and projects
6. Software and program development

1 Self-Organization

Reading: *Nonlinear Dynamics and Chaos*, Strogatz (NDAC), and Course Lecture Notes

Theme: Forms of Randomness, Order, and Intrinsic Instability

1. Nonlinear Dynamics:
   (a) Qualitative dynamics
   (b) ODEs and maps
   (c) Bifurcations
   (d) Stability, instability, and chaos
   (e) Quantifying (in)stability
2. Pattern-forming systems:
   (a) Instability and stabilization of patterns
   (b) Cellular automata, map lattices, spin systems

1.1 Lecture 2 (10 January): The Big Picture

Reading: *NDAC*, Chapters 1 and 2.

Topics:

1. Pendulum demo
2. Discuss *Chaos* and *Odds* readings and homework
3. Qualitative dynamics: A geometric view of behavior
4. State space
5. Flows
6. Attractors
7. Basins

8. Submanifolds
9. Concrete, but simple example: One-dimensional flows

Homework: Everyday unpredictability; see handout or website. Due in one week, but be prepared to discuss at next meeting.

1.2 Lecture 3 (15 January): Example Dynamical Systems

Reading: *NDAC*, Sections 6.0-6.7, 7.0-7.3, and 9.0-9.4.

Topics:

1. Continuous-time ODEs
   (a) 2D flows: Fixed points (Sec. 6.0-6.4)
   (b) 2D flows: Limit cycles (Sec. 7.0-7.3)
   (c) 3D flows: Chaos in Lorenz (Sec. 9.0-9.4)
   (d) Simulation demo
2. From continuous to discrete time (Sec. 9.4)
   (a) Poincaré maps and sections
   (b) Lorenz ODE to cusp map
   (c) Rössler ODE to logistic map (pp. 376–379)
   (d) Discrete-time maps

1.3 Lecture 4 (17 January): The Big, Big Picture (Bifurcations)

Reading: *NDAC*, Chapters 3 and 8 and Sec. 10.0-10.4.

Topics:

1. Qualitative dynamics: Space of all dynamical systems
2. Example: Bifurcations of one-dimensional flows
   (a) Saddle node
   (b) Transcritical
   (c) Pitchfork
3. Catastrophe theory
   (a) Catastrophes: Fixed point to fixed point bifurcation
   (b) Example: Cusp Catastrophe
   (c) Catastrophe theory classification of fixed point bifurcations
4. Bifurcations in discrete-time maps: Logistic map
5. Fixed point to limit cycle
6. Phenomenon and calculation
7. Limit cycle to limit cycle
8. Phenomenon and calculation
9. Routes to chaos: Period-doubling cascade
10. Phenomenon and calculation

11. Band-merging
12. Periodic windows and intermittency
13. Simulation demo

Homework: Collect Week 0's, assign this week's today.

1.4 Lecture 6 (22 January): Mechanism of Chaos: Stable Instability

Reading: *NDAC*, Sec. 12.0-12.3, 9.3, and 10.5.

Topics:

1. Chaotic mechanisms: Stretch and fold
2. Baker's map
3. Cat map (and stretch demo)
4. Henon map: stretch-fold and self-similarity
5. Roessler attractor branched manifold
6. Dot spreading: Roessler and Lorenz ODEs
7. Lyapunov characteristic exponents (LCEs)
8. Time to unpredictability
9. Dissipation rate
10. Attractor LCE classification
11. Chaos defined

1.5 Lecture 7 (24 January): Example Chaotic Maps (that you can analyze)

Reading: *NDAC*, Chapter 10.

Topics:

1. Shift map
2. LCEs for maps
3. Tent map
4. Logistic map
5. LCE view of period-doubling route to chaos
6. Period-doubling self-similarity
7. Renormalization group analysis of scaling

Homework: Collect Week 1's, assign this week's today.

2 From Determinism to Stochasticity

Reading: Lecture Notes.

Theme: Stochasticity and Measurement

1. Probability theory of Dynamical Systems
2. Stochastic Processes
3. Measurement Theory

2.1 Lecture 8 (29 January): Probability Theory of Dynamical Systems

Reading: Lecture Notes.

Topics:

1. Probability theory review
2. Dynamical evolution of distributions
3. Invariant measures
4. Examples

2.2 Lecture 9 (31 January): Stochastic Processes

Reading: Lecture Notes.

Topics:

1. Review last lecture.
2. Processes
3. Markov chains
4. Statistical equilibrium
5. Hidden Markov models
6. Examples: Fair coin, periodic, golden mean, even, and others

Homework: Collect Week 2's, assign this week's today.

2.3 Lecture 10 (5 February): Measurement Theory

Reading: Lecture Notes.

Topics:

1. Review last lecture.
2. State-space partitioning
3. Orbit and sequence spaces
4. Markov partitions
5. Generating partitions
6. Examples: 1D maps (Optional: 2D Cat map)

3 Information Processing

Reading: *Elements of Information Theory*, Cover and Thomas (EIT), and *Computational Mechanics Reader*, JPC (CMR)

Theme: Information, Uncertainty, and Memory

1. Entropies
2. Communication Channel (and coding theorems)
3. Mutual Information and Information metric
4. Excess Entropy
5. Transient Information
6. Connection to Dynamics: Entropy rate and LCEs

3.1 Lecture 11 (7 February): Entropies

Reading: *EIT*, Chapters 1 and 2.

Topics:

1. Motivation: Information $\neq$ Energy
2. Information as uncertainty and surprise
3. Information sources: Ignorance of forces or initial conditions, deterministic chaos, and ...?
4. Axioms for a measure of information
5. Entropy function
6. Convexity
7. Joint and Conditional Entropy
8. Mutual information
9. Examples

Homework: Collect Week 3's, assign Week 4's today.

3.2 Lecture 12 (12 February): Information in Processes

Reading: *EIT*, Sec. 5-5.4 and 8-8.5 and Chapter 4.

Topics:

1. Communication channels
2. Coding theorems
3. Entropy rates for Markov chains
4. Entropies for times series
5. Entropy convergence

3.3 Lecture 13 (14 February): Memory in Processes

Reading: *CMR* article RURO.

Topics:

1. Excess entropy
2. Examples
3. Transient information
4. Examples

Homework: Collect Week 4's, assign Week 5's today.

4 Natural Computation

Reading: *Computational Mechanics Reader*, JPC (CMR)

Theme: Causal Architecture of Dynamical Systems and Stochastic Quantum and Processes

1. Prediction and Learning
2. ϵ-Machines and Causal Architecture
3. Measures of Structural Complexity
4. How to Calculate
5. Complex Materials
6. Quantum Systems

4.1 Lecture 14 (19 February): The Learning Channel

Reading:

1. *CMR* article *RURO* (Intro) and Lecture Notes.
2. *CMR* article *Chance and Order*, Stanislaw Lem, New Yorker **59** (1984) 88–98.
3. *CMR* article *Revealing Order in the Chaos*, Mark Buchanan, New Scientist, 26 February 2005; available at `cse.ucdavis.edu/~chaos/news/`.

Topics:

1. The Learning Channel
2. The Prediction Game
3. Space of histories
4. Predictive equivalence relation
5. Causal states
6. ϵ-Machines

Projects: Project topic should be selected by now.

4.2 Lecture 15 (21 February): ϵ-Machines

Reading: *CMR* article *CMPPSS*.

Topics:

1. Examples: Predictable, fair coin, period-two
2. Optimal Prediction
3. Minimality
4. Uniqueness
5. Minimal Sufficient Statistic
6. Minimal Stochasticity

Homework: Collect Week 5's, assign this week's today.

4.3 Lecture 16 (26 February): Measures of Structural Complexity

Reading: *CMR* article *CMPPSS*.

Topics:

1. Entropy rate
2. Statistical complexity
3. Excess entropy bound
4. Examples: (hidden) Markov chains and dynamical systems

Material Covered: *NDAC* readings, *EIT* readings, and *CMR* article RURO.

Topics Covered:

1. Dynamics
2. Information Theory

4.4 Lecture 17 (28 February and 4 March): Complex Materials or ?

I am currently considering replacing the remaining lectures with new material on causal inference, rate distortion theory, and interactive learning.

Reading: *CMR* articles *BTFM1* and *BTFM2*.

Topics:

1. One-Dimensional materials: Physics of polytypes
2. Experimental studies
3. Fault model
4. ϵ-Machine spectral reconstruction
5. Structure in disorder: Beyond the fault model
6. Zinc-Sulfide

Homework: Week 6's due; assign Week 7's.

4.5 Lecture 18 (6, 11, and 13 March): Computation in Quantum Systems or ?

Reading: *CMR* article *CIFQP*.

Topics:

1. Stochastic languages and machines
2. Quantum languages
3. Quantum machines
4. Examples: Quantum fair coin, golden mean, even processes
5. Quantum dynamical systems: Iterated beam splitter and ion traps
6. Quantum computation

Homework: Week 7's due 6 March.

5 Project Presentations

1. Presentations will be organized according to class size.
2. If the class is large, most likely they will be given at a mini-workshop, some evening.

Note: Project write-ups due Friday 14 March.