

Natural Computation and Self-Organization:
The Physics of Information Processing in Complex Systems

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Homework 5

Covering *Lecture Notes* and *EIT* Second Edition, Chapters 1 and 2.

1. Write up your Project Proposal with the following sections. The result should be 1-2 pages long.
 - (a) Goal: What you would like to learn?
 - (b) System: Describe how the dynamical system is nonlinear and time-dependent. What's the state space? What's the dynamic? Why is it interesting?
 - (c) Properties: What properties are you going to investigate?
 - (d) Methods: What methods will you use? Why are they appropriate?
 - (e) Hypothesis: What is your current guess as to what you will find?
 - (f) Steps: List the appropriate steps for your investigation; for example, read literature, write simulator, do mathematical analysis, estimate properties from simulation, write up report, and so on.
 - (g) Time: Estimate how long each step will take. Can you complete the project within one month?
2. Consider the map of the interval given by

$$x_{n+1} = f(x_n) = \begin{cases} 1 - 2x_n & 0 \leq x_n \leq \frac{1}{3} \\ 2x_n - \frac{1}{3} & \frac{1}{3} < x_n \leq \frac{2}{3} \\ 3 - 3x_n & \frac{2}{3} < x_n \leq 1 \end{cases} \quad x_n \in [0, 1] . \quad (1)$$

Construct a Markov partition for this map, as follows.

- (a) Plot x_{n+1} versus x_n .
 - (b) Find the monotone pieces of $f(x)$ by determining the locations of its extrema.
 - (c) Show that the segments defined by the ends of the interval and these locations form a Markov partition.
 - (d) Give the Markov chain induced by this partition.
 - (e) Calculate $p_V(\infty)$ for the Markov chain.
3. Last week you constructed a 3 element Markov partition $\mathcal{P}_{\text{Markov}}$ for the logistic map at the parameter setting where 2 bands merge into 1 band. Analyze a generating partition for this same situation, as follows.
 - (a) Consider the binary partition of the interval $\mathcal{P}_{\text{gen}} = \{0 \sim (0, \frac{1}{2}), 1 \sim (\frac{1}{2}, 1)\}$. Label the transitions in last week's Markov chain with the symbols observed using \mathcal{P}_{gen} when making transitions between Markov partition elements.
 - (b) Show that the resulting representation is a deterministic hidden Markov process.
 - (c) How many internal (Markov) state sequences map onto the observed sequence of all 1s? Are there any other examples of many-to-one mappings of state sequences onto observed sequences?

- (d) States are equivalent if the sets of sequences that follow from them are the same. Give a hidden Markov chain with fewer states than the one just constructed by identifying states that are equivalent in this sense.
4. For the logistic map at $r = 4$, what sets of initial conditions lead to the sequences $s^3 = 001$ and $s^3 = 111$ when the map iterates are observed with the partition $\mathcal{P}_{\text{gen}} = \{0 \sim (0, \frac{1}{2}), 1 \sim (\frac{1}{2}, 1)\}$?
 5. *EIT* Problem 2.6. (*EIT* 1st edition: was Problem 2.10.)
 6. *EIT* Problem 2.12. (*EIT* 1st edition: was Problem 2.16.)
 7. *EIT* Problem 2.15. (*EIT* 1st edition: was Problem 2.21.)

Homework due one week after being assigned.